FLOW OF COUPLE STRESS FLUID IN CONTACT WITH A NEWTONIAN FLUID IN AN INCLINED CHANNEL BOUNDED BY PERMEABLE BEDS

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Abstract: The problem of steady laminar fully developed flow and heat transfer in an inclined channel consisting of a couple-stress fluid sandwiched between two Newtonian fluids with permeable beds is investigated. The flow in the permeable beds is governed by Darcy's law. The closed form solutions are obtained for the velocity and temperature fields in the channel. The effect of permeability on the velocity and temperature is discussed. We observed that for a given y, the velocity increases with increasing Darcy number and for a given y, the temperature increases with increasing couple stress parameter a.

Key words: couple-stress fluid, Newtonian fluid, inclined channel, permeable bed.

I. INTRODUCTION

The flow of non-Newtonian fluids through and past porous media have wider applications in many branches like petroleum Engineering, chemical engineering and such other important fields. There are several non Newtonian fluid models available in literature. Some of these models deal with theory of polar fluids. The theory of polar fluids was derived from a statistical mechanics model that assumed non central forces of interaction between particles. If the inter particle forces are not central forces in a particle interaction, there is an inter particle couple as well as an inter particle force. Due to action of this couple, the fluid particles will have a tendency to rotate relative to

their neighbors. The idea of polar fluid is obtained by introducing a kinematic variable to model the forces that balance the action of the couple.

The couple stress fluid model is very useful to explain the behaviour of some slurries and some physiological fluids. Initial studies of experimental data on blood suggest that some derivations from Newtonian behaviour of blood may be explained through couple stress fluid model. Based on the couple stress theory of Stokes (1966), Chaturani and Kaloni (1976), Chaturani and Upadthya (1979) have studied some theoretical models for blood flow through narrow tubes.

Chaturani et.al., (1981) studied a three layered coquette flow model for blood flow and they assume that the top and bottom layers consist of plasma (Newtonian fluid) and the middle consist of a red cell suspension couple stress fluid. Umavathi and Malashetty (1999) studied the effects of couple stresses on the free convective flow in a vertical channel. Free convection flow of an electrically conducting couple-stress fluid for the radiating medium in a vertical channel has been studied by Umavathi (2000).T.K.V.Iyengar et.al, (2011) studied the pulsating flow of an incompressible couple stress fluid between permeable beds.

Umavathi et.al., (2005) made a detailed study on the flow of heat transfer of a couple stress fluids in contact with a Newtonian fluid. Keeping in view the practical applications described above we have analyzed laminar fully developed flow and heat transfer in an inclined channel consisting of a couple stress fluid sandwiched between two Newtonian fluids bounded by permeable beds. The flow in the permeable beds is governed by Darcy's law. The closed from solutions are obtained for the velocity and temperature fields in the channel. The effect of permeability on the velocity and temperature is discussed.

II. NOMENCLATURES

x, y	-	Cartesian coordinates
u_1, u_2, u_3	-	Velocity in region I, region II, region III
U_{B1}	-	Slip velocity at the upper permeable bed
U_{B2}	-	Slip velocity at the lower permeable bed
Da_1	-	Darcy number at the upper permeable

		bed $\left(\frac{k_1}{h^2}\right)$
Da_2	-	Darcy number at the lower permeable bed $\frac{k_2}{h^2}$
T_{B1}	-	Temperature at $y = 2h$
T_{B2}	-	Temperature at $y = -h$
θ	-	Angle of inclination of the channel with the horizontal.
α	-	Slip parameter
$\overline{u_1}$	-	Average velocity
a	-	Couple stress parameter $\left(\frac{\mu_1 h^2}{\eta}\right)$
C_p	-	Specific heat at constant pressure
Ec	-	Eckert number
h	-	Hight of the regions , I, II, III
K	-	Ratio of thermal conductivities $\frac{k_1}{k_2}$
K_1	-	Thermal conductivity of the fluid in regions I and III
K_2	-	Thermal conductivity of the fluid in region II
m	-	Ratio of viscosities $\frac{\mu_1}{\mu_2}$
P_1	-	Non dimensional pressure gradient , $-\mathrm{Re}\frac{\partial p}{\partial x}$
P	-	$p_1 - \frac{\mathrm{Re}}{F}\sin\theta$
T_1, T_2, T_3	-	Temperatures in region I, II and III
Pr	-	Prandtl number

III MATHEMATICAL FORMULATION OF THE PROBLEM:

Consider the steady laminar flow of a couple stress fluids in contact with a Newtonian fluid in a channel bounded permeable beds as shown in figure 1.

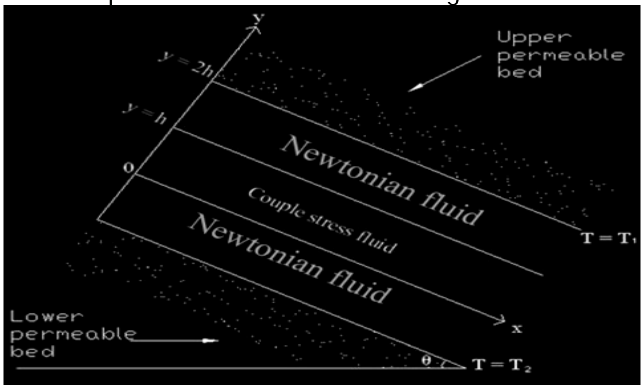

Fig 1: Physical model

The channel is inclined at an angle θ with the horizontal. The regions $-h \le y \le 0$ and $h \le y \le 2h$ are filled with a clear viscous fluid of viscosity μ_1 and thermal conductivity k_1 and the region $0 \le y \le h$ is occupied by a couple stress fluid viscosity μ_2 and thermal conductivity k_2. The fluids in all regions are assumed to be immiscible and the transport properties of the fluids in all regions are assumed to be constant the flow in the permeable beds is described by Darcy's law.

The following assumptions are made in simplifying the basic equations

i. The flow is steady, viscous, incompressible and fully developed.
ii. The flow is in x-direction. All the physical quantities, except the pressure are functions of y only.
iii. The motion is caused by a constant pressure gradient $\frac{\partial p}{\partial x}$.
iv. The permeability's of the lower and upper beds are k_1 and k_2.

In view of the above assumptions, the equations of motion for velocity take the following form:

Flow in the channel
Basic equations

$$\frac{\partial u}{\partial x} = 0 \qquad (1)$$

Region I $\qquad (h \le y \le 2h)$

$$\mu_1 \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial p}{\partial x} + \rho g_x = 0 \qquad (2)$$

$$-\frac{\partial p}{\partial y} + \rho g_y = 0$$

Region II $\qquad (0 \le y \le h)$

$$\mu_2 \frac{\partial^2 u_2}{\partial y^2} - \frac{\partial^4 u_2}{\partial y^4} - \frac{\partial p}{\partial x} + \rho g_x = 0 \qquad (3)$$

$$-\frac{\partial p}{\partial y} + \rho g_y = 0$$

Region III $\qquad (-h \le y \le 0)$

$$\mu_3 \frac{\partial^2 u_3}{\partial y^2} - \frac{\partial p}{\partial x} + \rho g_x = 0$$

$$-\frac{\partial p}{\partial y}+\rho g_y=0 \qquad (4)$$

Similarly the equations of motion for temperature in the three regions take the following form:

Region I $\quad (h \le y \le 2h)$

$$\frac{\partial^2 T_1}{\partial y^2}+C\frac{\mu_1}{k_1}\left(\frac{\partial u_2}{\partial y}\right)^2=0 \qquad (5)$$

Region II $\quad (0 \le y \le h)$

$$\frac{\partial^2 T_2}{\partial y^2}+C\frac{\mu_2}{k_2}\left(\frac{\partial u_2}{\partial y}\right)^2=0 \qquad (6)$$

Region III: $\quad (-h \le y \le 0)$

$$\frac{\partial^2 T_3}{\partial y^2}+C\frac{\mu_3}{k_3}\left(\frac{\partial u_3}{\partial y}\right)^2=0 \qquad (7)$$

Flow in the porous medium

The flow in the lower and upper beds is governed by the following Darcy's law.

$$Q_i=\frac{k_i}{\mu}\frac{\partial p}{\partial x}-\rho g_x$$

Where
$i=1$ for upper permeable bed and
$i=2$ for lower permeable bed

Boundary Conditions

$$U_1=U_{B1},\quad \frac{\partial u_1}{\partial y}=\frac{-\alpha}{\sqrt{k_1}},\quad \left(U_{B1}-\overline{Q_1}\right)\ at\ y=2h \qquad (8)$$

$$U_1=U_2 \quad at \quad y=h \qquad (9)$$

$$U_2=U_3 \quad at \quad y=0 \qquad (10)$$

$$U_3=U_{B2},\ \frac{\partial u_3}{\partial y}=\frac{\alpha}{\sqrt{k_2}}\left(U_{B2}-Q_2\right) \quad at \quad y=-h \qquad (11)$$

$$\mu_1\frac{\partial u_1}{\partial y}=\mu_2\frac{\partial u_2}{\partial y}-\eta\frac{\partial^3 u_2}{\partial y^3} \quad at \quad y=h \qquad (12)$$

$$\mu_2\frac{\partial u_2}{\partial y}-\eta\frac{\partial^3 u_2}{\partial y^3}=\mu_1\frac{\partial u_3}{\partial y} \quad at \quad y=0 \qquad (13)$$

$$\frac{\partial^2 u_2}{\partial y^2}=0 \quad at \quad y=h \qquad (14)$$

$$\frac{\partial^2 u_2}{\partial y^2}=0 \quad at \quad y=0 \qquad (15)$$

$$T_1=T_{B1},\quad \frac{\partial T}{\partial y}=\frac{-H}{\sqrt{k_1}}\left(T_{B1}-T_0\right) \quad at \quad y=2h \qquad (16)$$

$$T_1=T_2 \quad at \quad y=h \qquad (17)$$

$$T_2=T_3 \quad at \quad y=0 \qquad (18)$$

$$T_3=T_{B2},\frac{\partial T}{\partial y}=\frac{H}{\sqrt{k_2}}\left(T_{B2}-T_0\right) \quad at \quad y=-h \qquad (19)$$

$$K_1\frac{\partial T_1}{\partial y}=K_2\frac{\partial T_2}{\partial y} \quad at \quad y=h \qquad (20)$$

$$K_2\frac{\partial T_2}{\partial y}=K_3\frac{\partial T_3}{\partial y} \quad at \quad y=0 \qquad (21)$$

IV. NON-DIMENSIONALIZATION OF THE FLOW QUANTITIES:

Introduced to make the basic equations and the boundary conditions dimensionless:

$$x^*=\frac{x}{h},\qquad y^*=\frac{y}{h},\qquad u_i^*=\frac{u_i}{\overline{u}}\quad (i=1,2)$$

$$Q_i^*=\frac{Q_i}{u_1},\qquad \theta=\frac{T-T_{B2}}{T_{B1}-T_{B2}};\qquad m=\frac{\mu_1}{\mu_2};\qquad p^*=\frac{p}{\rho u^2}$$

In view of the above dimensionless quantities the basic equations and the boundary conditions for the velocity take the following form (Neglecting the bars).

Basic Equations
Region I:

$$\frac{d^2 u_1}{dy^2}=H_1 \qquad (22)$$

$$\frac{\partial p}{\partial y}+\frac{\cos\theta}{F}=0 \qquad (23)$$

Region II:

$$\frac{d^4 u_2}{dy^4}-\frac{a^2}{m}\frac{d^2 u_2}{dy^2}=H_2 \qquad (24)$$

$$\frac{\partial p}{\partial y}+\frac{\cos\theta}{F}=0 \qquad (25)$$

where $H_2=\frac{\mathrm{Re}}{F}a^2\sin\theta-a^2P$

Region III:

$$\frac{d^2 u_1}{dy^2}=H_1 \qquad (26)$$

$$\frac{\partial p}{\partial y}+\frac{\cos\theta}{F}=0 \quad (27)$$

Where $H_1=P_0-\frac{\mathrm{Re}}{F}\sin\theta$, $P_0=\mathrm{Re}\frac{\partial p}{\partial x}$, $F=\frac{gh}{u^2}$

Boundary conditions are

$$U_1=U_{B1} \quad at \quad y=2 \quad (28)$$

$$\frac{du_1}{dy}=\frac{-\alpha}{\sqrt{Da_1}}\left(U_{B1}-Q_1\right) \quad at \quad y=2 \quad (29)$$

$$U_1=U_2 \quad at \quad y=1 \quad (30)$$

$$U_2=U_3 \quad at \quad y=0 \quad (31)$$

$$U_3=U_{B2} \quad at \quad y=-1 \quad (32)$$

$$\frac{du_3}{dy}=\frac{\alpha}{\sqrt{Da_2}}\left(U_{B2}-Q_2\right) \quad at \quad y=-1 \quad (33)$$

$$\frac{du_1}{dy}=\frac{1}{m}\frac{du_2}{dy}-\frac{1}{a^2}\frac{d^3u_2}{dy^3} \quad at \quad y=1 \quad (34)$$

$$\frac{du_2}{dy}=\frac{m}{a^2}\frac{d^3u_2}{dy^3}=m\frac{du_3}{dy} \quad at \quad y=0 \quad (35)$$

$$\frac{d^2u_2}{dy^2}=0 \quad at \quad y=1 \quad (36)$$

$$\frac{d^2u_2}{dy^2}=0 \quad at \quad y=0 \quad (37)$$

Similarly the basic equations and the boundary conditions for the temperature take the following form.

Basic equations

Region I:

$$\frac{d^2\theta_1}{dy^2}+Ec\,\mathrm{Pr}\left(\frac{du_1}{dy}\right)^2=0 \quad (38)$$

Region II

$$\frac{d^2\theta_2}{dy^2}+\frac{K}{m}Ec\,\mathrm{Pr}\left(\frac{du_2}{dy}\right)^2=0 \quad (39)$$

where $K=\frac{k_1}{k_2}$

Region III:

$$\frac{d^2\theta_3}{dy^2}+Ec\,\mathrm{Pr}\left(\frac{du_3}{dy}\right)^2=0 \quad (40)$$

Boundary conditions are

$$\frac{d\theta_1}{dy}=\frac{-H}{\sqrt{Da_1}}\theta_{B1} \quad at \quad y=2 \quad (41)$$

$$\theta_1=\theta_2 \quad at \quad y=1 \quad (42)$$

$$\theta_2=\theta_3 \quad at \quad y=0 \quad (43)$$

$$\frac{d\theta_3}{dy}=\frac{H}{\sqrt{Da_2}}\theta_{B2} \quad at \quad y=-h \quad (44)$$

$$\frac{d\theta_1}{dy}=\frac{1}{K}\frac{d\theta_2}{dy} \quad at \quad y=h \quad (45)$$

$$\frac{d\theta_2}{dy}=K\frac{d\theta_3}{dy} \quad at \quad y=0 \quad (46)$$

V. SOLUTION

Solving the equations (21) – (23) subject to the boundary conditions (27) – (36), we obtain the velocity field as

$$U_1=\frac{Py^2}{2}+c_1y+c_2,$$ Where

$$p=\frac{h^2}{\mu_1 u_1}\left(\frac{\partial p}{\partial x}-\rho g_x\right) \quad 1\le y\le 2 \quad (47)$$

$$U_2=c_3+c_4y+c_5\cosh(A_4y)+c_6\sinh(A_4y)+A_5y^2, 0\le y\le 1 \quad (48)$$

$$U_3=\frac{py^2}{2}+c_7y+c_8 \quad -1\le y\le 0 \quad (49)$$

Where $A_5=\frac{-G_2}{2A_2}$, $A_2=\frac{a^2}{m}$, $A_4=\sqrt{A_2}$, $G_2=-a^2p$

$$C_1=\frac{mc_7+2A_5-mp}{m},\ C_2=\frac{mB_3-B_4(mc_7+2A_5-mp)}{m}$$

$$C_3=c_8-c_5,\ C_4=mc_7,\ C_5=\frac{-2A_5}{A_4^2}$$

$$C_6=-C_5D_1,\quad C_7=\frac{m^2C_7+mD_2-mB_1-mC_5-B_5}{m(1-B_2-B_4)},$$

$$C_8=B_2C_7-B_1,\quad D_1=\frac{\cosh A_4-1}{\sinh A_4}$$

$$D_2=C_5\cosh A_4+C_6\sinh A_4+2A_5-P/2$$

$$B_1=\left(\frac{1}{2}+\frac{\sqrt{Da_2}}{\alpha}\right)p.Q_2\ B_2=\left(1+\frac{\sqrt{Da_2}}{\alpha}\right),$$

$$B_3=Q_1-2p\left(1+\frac{\sqrt{Da_1}}{\alpha}\right),\qquad B_4=\left(2+\frac{\sqrt{Da_1}}{\alpha}\right)$$

$$B_5=2A_5-mp+mB_3+mB_4P-2B_4A_5,$$

The slip velocity at the upper permeable bed is obtained as

$$U_{B1} = 2C_1 + C_2 + 2P \qquad (50)$$

The slip velocity at the lower permeable bed is obtained as

$$U_{B2} = -C_7 + C_8 + \frac{P}{2} \qquad (51)$$

Solving the equations (38)–(40) subject to the boundary conditions (41)–(46) we obtain the temperature as

$$\theta_1 = \alpha_1 y^4 + \alpha_2 y^3 + \alpha_3 y^2 + B_1 y + B_2 \qquad (52)$$

$$\theta_2 = \alpha_{14}\cosh(2A_4 y) + \alpha_{15}\sinh(2A_4 y) + \alpha_{16} y\cosh(A_4 y)$$

$$+\alpha_{17} y\sinh(A_4 y) + \alpha_{18}\cosh(A_4 y) + \alpha_{19}\sinh(A_4 y)$$
$$+\alpha_{20} y^4 + \alpha_{21} y^3 + \alpha_{22} y^2 + B_3 y + B_4 \qquad (53)$$

$$\theta_3 = \alpha_{23} y^4 + \alpha_{24} y^3 + \alpha_{25} y^2 + B_5 y + B_6 \qquad -1 \le y \le 0 \ (54)$$

Where

$$\alpha_1 = -\frac{Ec\Pr P^2}{12}, \qquad \alpha_2 = -\frac{Ec\Pr \rho c_1}{3}$$

$$\alpha_3 = -\frac{Ec\Pr \rho c_1^2}{2} \qquad \alpha_4 = A_6 A_4^2 C_5^2$$

$$\alpha_6 = A_6 C_5 C_6 A_4^2 \qquad \alpha_7 = 4A_6 A_4 A_5 C_6$$

$$\alpha_8 = 4A_6 A_5 A_4 C_5 \qquad \alpha_9 = 2A_6 C_4 C_6 A_4$$

$$\alpha_{10} = 2A_6 C_4 C_5 A_4 \qquad \alpha_{11} = 4A_6 A_5^2$$

$$\alpha_{12} = 4A_6 A_5 C_4 \qquad \alpha_{13} = A_6 C_4^2$$

$$\alpha_{14} = \frac{(\alpha_4 + \alpha_5)}{8A_4^2} \qquad \alpha_{15} = \frac{\alpha_6}{4A_4^2}$$

$$\alpha_{16} = \frac{\alpha_7}{4A_4^7} \qquad \alpha_{17} = \frac{\alpha_8}{A_4^2}$$

$$\alpha_{18} = \left(\frac{-2\alpha_8}{A_4^3} + \frac{\alpha_9}{A_4^2}\right) \qquad \alpha_{19} = \left(\frac{-2\alpha_7}{A_4^3} + \frac{\alpha_{10}}{A_4^2}\right)$$

$$\alpha_{20} = \frac{\alpha_{11}}{12} \qquad \alpha_{21} = \frac{\alpha_{12}}{6}$$

$$\alpha_{22} = \frac{2\alpha_{13} + \alpha_5 - \alpha}{4} \qquad \alpha_{23} = \frac{-Ec\Pr P^2}{12}$$

$$\alpha_{24} = \frac{-Ec\Pr P C_7}{3} \qquad \alpha_{25} = \frac{-Ec\Pr c_7^2}{2}$$

$$T_{B2} = -B_5 + B_6 + e_2 \qquad e_1 = -4\alpha_{23} + 3\alpha_{24} - 2\alpha_{25}$$

$$e_2 = \alpha_{23} - \alpha_{24} + \alpha_{25}$$

$$B_6 = \left(\frac{\sqrt{Da_2}}{H} + 1\right)B_5 + \frac{\sqrt{Da_2}}{H}$$

$$T_{B1} = -\frac{\sqrt{Da_1}}{H}(32\alpha_1 + 1$$

$$e_3 = 32\alpha_1 + 12\alpha_2 + 4\alpha_3$$

$$e_4 = 16\alpha_1 + 8\alpha_2 + 4\alpha_3 \qquad e_5 = 2A_4\alpha_1^5 + \alpha_{16} + A_4\alpha_{19}$$

$$e_6 = 2A_4\alpha_{14}\sinh 2A_4 + 2A_4\alpha_{15}\cosh 2A_4$$
$$+A_4\alpha_{16}\sinh A_4 + A_4\alpha_{18}\sinh A_4 + A_4\alpha_{17}$$
$$\cosh A_4 + A_4\alpha_{19}\cosh A_4 + \alpha_{16}\cosh A_4$$
$$+\alpha_{17}\sinh A_4 + 4\alpha_{20} + 3\alpha_{21} + 2\alpha_{22}$$

$$e_7 = 4\alpha_1 + 3\alpha_2 + 2\alpha_3, \qquad e_8 = \alpha_{14} + \alpha_{18}$$

$$e_9 = \alpha_{14}\cosh 2A_4 + \alpha_{15}\sinh 2A_4 + \alpha_{16}\cosh A_4$$
$$+\alpha_{18}\cosh A_4 + \alpha_{17}\sinh A_4$$
$$+\alpha_{20} + \alpha_{21} + \alpha_{22} - \alpha_1 - \alpha_2 - \alpha_3$$

VI. DEDUCTIONS AND DISCUSSIONS

From equations (1) – (4) we have calculated velocities U (i.e. U_1, for region I, U_2 for region II, and U_3 for region III) as function of y, for different values of Darcy number Da for fixed $P = -5,\ a = 1,\ m = 2,\ \alpha = 0.3$ and is shown in figure 2. We observe that the velocity increases with the increase in y initially and then it decreases with the increment in y. For a given y, we notice that the velocity increases with increasing Darcy number.

The variation of velocity U with y, are calculated from equations (1)-(4) for different values of couple stress parameter a and is shown in figure 3. We observe that for a given y, the velocity increases with increasing couple stress parameter a.

The variations of U1, U2 and U3 with y are calculated from equations (1)-(4) for different values of viscosity ratio and are shown in figure 4. We observe that for a given y, the velocity decreases with the increasing in the viscosity ratio m.

From equation (52)-(54) we have calculated temperature θ (θ_1 for Region I, θ_2 for Region II, and θ_3 for Region III) as a function of y for different values of Darcy number and for fixed $P = -5, a = 1, m = 2, \alpha = 0.3, K = 1, E = 1, Da = 0.1$ and $H = 0.6$ and is shown in figure 5. We observe that the temperature increases with the increasing in y and then it decreases with the increment in y. For a given y, we notice that the temperature increases with an increase in the Darcy number.

The variation of temperature θ with y are calculated from equations (52)-(54) for different values of couple stress parameter and shown in figure 6, for fixed $P=-5, a=1, m=2, \alpha=0.3, K=1, \in=1, Da=0.1, H=0.6$. We observe that for a given y, and temperature increase with increasing couple stress parameter a. The variation of temperature θ with y are calculated from equations (52)-(54) for different values of ratio of viscosities m and is shown in figure 7, for fixed $P=-5, a=1, m=2, \alpha=0.3, K=1, \in=1, Da=0.1, H=0.6$. We observe that for a given y, the temperature decreases with increasing viscosity ratio m.

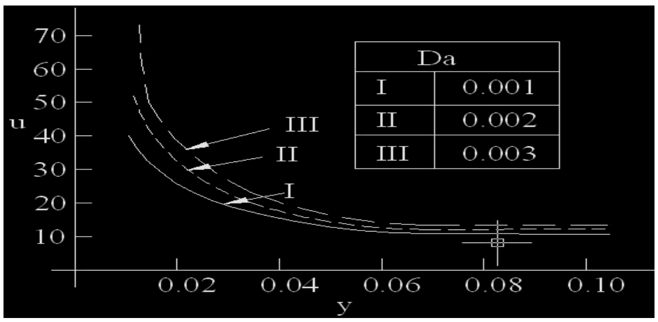

Fig. 2 Velocity profiles for different values of Da with fixed values of p=-5, a=1, m=2, α=0.3

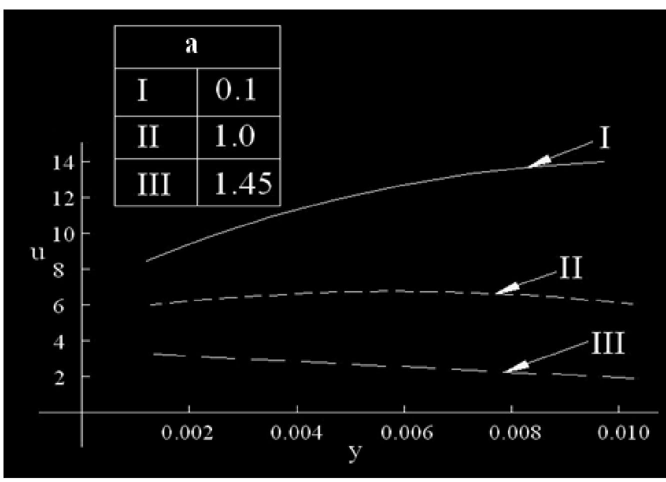

Fig. 3 Velocity profiles for different values of 'a' with fixed values of p=-5, a=1, Da=0.1, m=2

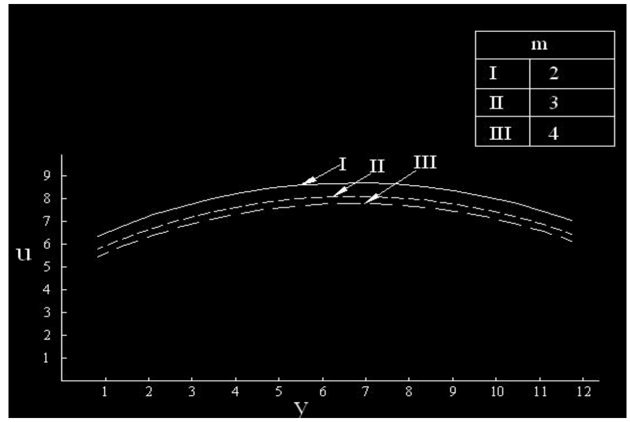

Fig. 4 Velocity profiles for different values of m with fixed values of p=-5, a=1, α =0.3, Da=0.1

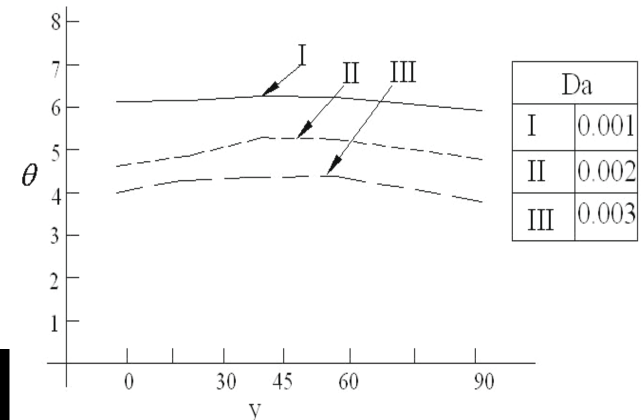

Fig.5 Temperature profiles for different values ofDa with fixed values of p=-5, m=2, K=1, H=0.6, $\alpha=0.3$

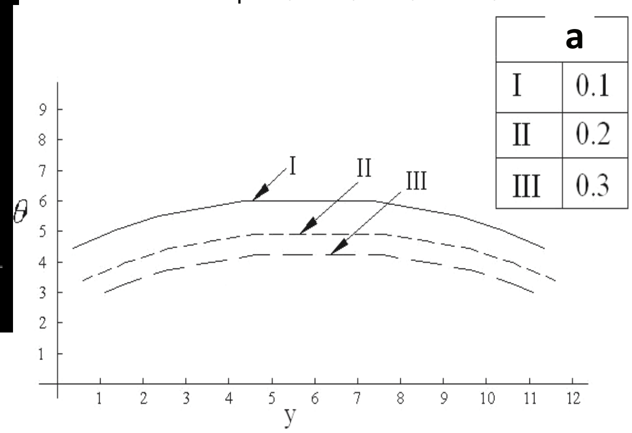

Fig. 6 Temperature profiles for different values of **'a'** with fixed values of p=-5, m=2,Da=0.1,K=1,H=0.6.

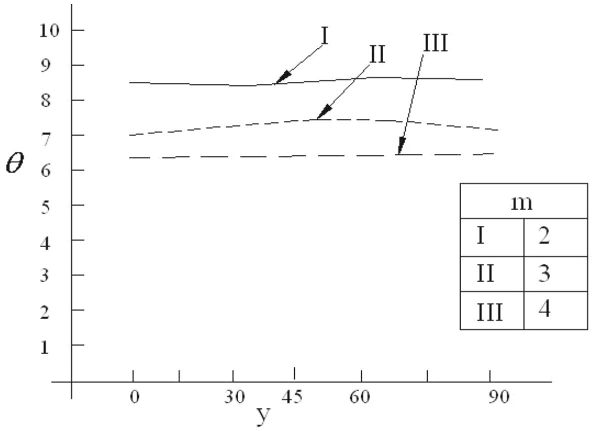

Fig.7 Temperature profiles for different values of m with fixed values of p=-5, Da=0.1, K=1, H=0.6, $\alpha = 0.3$.

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