Pulsatile flow of a Jeffrey fluid in a channel bounded by porous lined plates with suction and injection

S.Sreenadh[1] P.Govardhan[1] Y.V.K.Ravi Kumar[2]
[1] Department of Mathematics, S. V. University, Tirupati – 517502, INDIA
[2] Practice School Division, Birla Institute of Technology and Science (BITS) – Pilani, INDIA (yvkravi@gmail.com , corresponding author)

Abstract— Pulsatile flow of a Jeffrey fluid in a channel bounded by porous lined plates with suction and injection is studied in this paper. The steady and unsteady velocities are obtained. The effect of various parameters on the flow phenomenon is discussed through graphs..

***Key words* — pulsatile flow, Jeffrey fluid, suction, injection.**

I. INTRODUCTION

Viscous fluid flow through and past porous media is attracting the attention of scientists and engineers because of its wider applications in various branches of science and technology. The movement of ground water in soil, the seepage of water through earth fills and concrete dams, the movement of oil fields can described using the knowledge of flow through porous media. The petroleum industry has been showing a lot of interest in these problems in connection with the crude oil production from the under ground reservoirs. These reservoirs consists of porous materials like lime stone and dolomite where oil is preserved. Oil can be obtained by drilling wells down into the reservoir. In order to have a better oil production, it is necessary to use the knowledge of flow through porous media.

The oil available in the porous reservoir is a complex fluid. The properties of such fluid have impact on the oil production. The behavior of the oil may be Newtonian or non-Newtonian. In view of this, it is interesting to study non- Newtonian fluid flow through and past porous media. Further there are many important applications in biomechanics also (vide Fung and Tang, [1,2].

Muakat [3] made theoretical and experimental studies on porous flow using Darcy law. Darcy law is observed to be valid for low speed flows and agrees with several experiments modeled in one dimensional motion. Yih [4] suggested the modified Darcy law for describing unsteady flow through porous media. Following this law, Rudraiah et al. [5], analyzed several time dependent flows through and past permeable beds. Radhakrishnamacharya [6] investigated the pulsatile flow of a dusty fluid containing small solid particles uniformly, through a two-dimensional constricted channel. The effect of non-Newtonion nature of blood and pulsatality on flow through a stenosed tube is analyzed by Chaturani and Samy [7].

The pulsatile flow in a porous channel is important in understanding the process of dialysis of blood in an artificial kidney. Recently, Chandra and Prasad [8] discussed the pulsatile flow problems with periodic acceleration and varying cross section of tubes.

Wang [9] studied the interesting problem of pulsatile flow in a porous channel bounded by rigid walls. The pulsatile flow between permeable walls is important in understanding the blood flow in the circulatory system where the nutrients are supplied to tissues of various organs and waste products are removed. Vajravelu et al.,[10]made a detailed study on pulsatile flow between permeable beds. Avinash et.al [11] studied the pulsatile flow of a viscous stratified fluid of variable viscosity between permeable beds is studied. The interaction of peristaltic flow with pulsatile flow through a porous medium is discussed by Afifi and Gad [12].

In this paper an exact solution for the pulsatile flow of Jeffrey fluid in porous lined plates is obtained. The flow between the permeable layers is governed by Jeffrey model where as the flow in the lower and upper beds are governed by Darcy law. The velocity distributions in the porous and non-porous regions are determined.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider the pulsatile flow of a Jeffrey fluid in a channel bounded by porous lined plates (Fig.1). The thickness of the porous lining on each of the plates is ϵ. The fluid is injected into the channel from the lower porous layer with a velocity V and is sucked out into the upper porous layer with the same velocity. The permeabilities of lower and upper beds are k_1 and k_2 . The flow between the permeable beds is governed by the Jeffrey model whereas the flow in the porous medium is described by modified Darcy's law

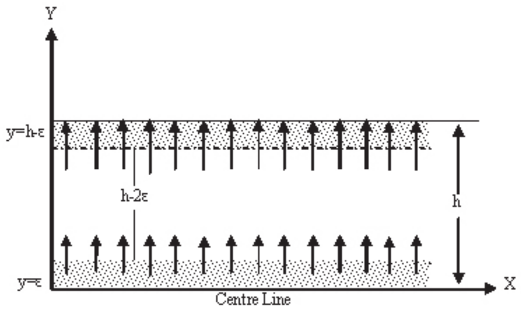

Fig.1. Physical Model

The following assumptions are made in the analysis of the problem

The flow is laminar and fully developed

The permeable beds are homogeneous

The flow is driven by unsteady pressure gradient. We assume that $\frac{1}{\rho}\frac{\partial p}{\partial x} = A + Be^{i\omega t}$

where A and B are constants and ' ω' is the frequency. In view of the above assumptions, the basic equations and boundary conditions of the flow take the following form.

Basic equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial y} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \frac{\upsilon}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

$$\frac{\partial p}{\partial y} = 0 \qquad (3)$$

Boundary conditions

$u = u_{B1}$ **at y = ∈** **(4)**

$\frac{\partial u}{\partial y} = \frac{\alpha}{\sqrt{k_1}}(u_{B1} - Q_1)$ **at y = ∈** **(5)**

$u = u_{B2}$ **at y = h - ∈** **(6)**

$\frac{\partial u}{\partial y} = \frac{-\alpha}{\sqrt{k_2}}(u_{B2} - Q_2)$ **at y = h - ∈** **(7)**

u, v are velocities of the fluid, ν is the coefficient of viscosity

p is the pressure, α is the slip parameter, k_1, k_{2a} are permeabilities of the lower and upper permeable beds, u_{B1}, u_{B2} are slip velocities in lower and upper permeable beds and λ_1 is the Jeffrey parameter.

Separating equations (1) – (7) into steady part denoted by a bar (-) and unsteady part denoted by a tilde (~) we get

Steady part:

$$\frac{\partial \bar{u}}{\partial x} = 0 \qquad \textbf{(8)}$$

$$V\frac{\partial \bar{u}}{\partial y} = -A + \frac{\upsilon}{1+\lambda_1}\upsilon\frac{\partial^2 \bar{u}}{\partial y^2} \qquad \textbf{(9)}$$

$\bar{u} = \bar{u}_{B1}$ **at y = ∈**

(10)

$\frac{d\bar{u}}{dy} = \frac{\alpha}{\sqrt{k_1}}(\bar{u}_{B1} - \bar{Q}_1)$ **at y = ∈**

(11)

$\bar{u} = \bar{u}_{B2}$ **at y = h - ∈** **(12)**

$\frac{d\bar{u}}{dy} = \frac{\alpha}{\sqrt{k_2}}(\bar{u}_{B2} - \bar{Q}_2)$ **at y = h - ∈** **(13)**

where $\bar{Q}_1 = \frac{k_1 \rho A}{\mu}(1+\lambda_1)$, $\bar{Q}_2 = \frac{k_2 \rho A}{\mu}(1+\lambda_1)$

Unsteady part:

$$\frac{\partial \tilde{u}}{\partial x} = 0 \qquad \textbf{(14)}$$

$$\frac{\partial \tilde{u}}{\partial t} + V\frac{\partial \tilde{u}}{\partial y} = -Be^{i\omega t} + \frac{\upsilon}{1+\lambda_1}\frac{\partial^2 \tilde{u}}{\partial y^2} \qquad \textbf{(15)}$$

$\tilde{u} = \tilde{u}_{B1}$ **at y = ε** **(16)**

$\frac{d\tilde{u}}{dy} = \frac{\alpha}{\sqrt{k_1}}(\tilde{u}_{B1} - \tilde{Q}_1)$ **at y = ε** **(17)**

$\tilde{u} = \tilde{u}_{B2}$ **at y = h - ε** **(18)**

$$\frac{d\tilde{u}}{dy} = \frac{-\alpha}{\sqrt{k_2}}(\tilde{u}_{B2} - \tilde{Q}_2) \quad \text{at y = h – ε} \quad (19)$$

where $\tilde{Q}_1 = \frac{-k_1\rho B(1+\lambda_1)}{\mu}e^{i\omega t}$ **,** $\tilde{Q}_2 = \frac{-k_2\rho B(1+\lambda_1)}{\mu}e^{i\omega t}$

Non – Dimensionalization of flow quantities

The following non – dimensional quantities are introduced to make the basic equations and the boundary conditions dimensionless

Steady part:

$$\bar{u}^* = \frac{\bar{u}}{(\frac{A_1 h}{v})}, \quad \bar{u}_{B1}^* = \frac{\bar{u}_{B1}}{(\frac{A_1 h}{v})}, \quad \bar{u}_{B2}^* = \frac{\bar{u}_{B2}}{(\frac{A_1 h}{v})} \quad x^* = \frac{x}{h},$$

$$y^* = \frac{y}{h}, \bar{Q}_1^* = \frac{\bar{Q}_1}{(\frac{A_1 h}{v})}, \bar{Q}_2^* = \frac{\bar{Q}_2}{(\frac{A_1 h}{v})},$$

$\varepsilon^* = \frac{\varepsilon}{h}$ **where** $A_1 = -A$.

Unsteady part:

$$\tilde{u}^* = \frac{\tilde{u}}{(\frac{h^2 B_1}{\upsilon})} \quad \tilde{u}_{B1}^* = \frac{\tilde{u}_{B1}}{(\frac{h^2 B_1}{\upsilon})} \quad \tilde{u}_{B2}^* = \frac{\tilde{u}_{B2}}{(\frac{h^2 B_1}{\upsilon})}, \tilde{t}^* = \frac{t}{(\frac{h^2}{\upsilon})},$$

$$\tilde{\omega}^* = \frac{\tilde{\omega}}{(\frac{\upsilon}{h^2})}, \quad \tilde{Q}_1^* = \frac{\tilde{Q}_1}{(\frac{h^2 B_1}{\upsilon})}$$

$\tilde{Q}_2^* = \frac{\tilde{Q}_2}{(\frac{h^2 B_1}{\upsilon})}$, $y^* = \frac{y}{h}, \varepsilon^* = \frac{\varepsilon}{h}$ **where** $B_1 = -B$.

In view of the above dimensionless quantities Eqs.(8) to (19) take the following form, neglecting the asterisks (*), we get,

Steady part

$$\frac{d\bar{u}}{dx} = 0 \quad (20)$$

$$\frac{d^2\bar{u}}{dy^2} - (1+\lambda_1)R\frac{d\bar{u}}{dy} = -(1+\lambda_1)R \quad (21)$$

$$\bar{u} = \bar{u}_{B1} \quad \text{at} \quad y = \varepsilon \quad (22)$$

$$\frac{d\bar{u}}{dy} = \alpha\sigma_1(\bar{u}_{B1} - \frac{1+\lambda_1}{\sigma_1^2}R) \quad \text{at } y = \varepsilon \quad (23)$$

$$\bar{u} = \bar{u}_{B2} \quad \text{at } y = 1 - \varepsilon \quad (24)$$

$$\frac{d\bar{u}}{dy} = -\alpha\sigma_2(\bar{u}_{B2} - \frac{1+\lambda_1}{\sigma_2^2}R) \quad \text{at } y = 1 - \varepsilon \quad (25)$$

where $R = \frac{Vh}{\upsilon}$ **is the Reynolds number,** $\sigma_1 = \frac{h}{\sqrt{k_1}}$ **,**

$\sigma_2 = \frac{h}{\sqrt{k_2}}$ **(**σ_1, σ_2 **are dimensionless parameters).**

Unsteady Part:

$$\frac{d\tilde{u}}{dx} = 0 \quad (26)$$

$$\frac{d^2\tilde{u}}{dy^2} - R(1+\lambda_1)\frac{d\tilde{u}}{dy} - (1+\lambda_1)\frac{d\tilde{u}}{dt} = -(1+\lambda_1)e^{i\omega t} \quad (27)$$

Letting $\tilde{u} = \tilde{f}(y)e^{i\omega t}$

$$\tilde{f}''(y) - R(1+\lambda_1)\tilde{f}'(y) - i\omega(1+\lambda_1)\tilde{f}(y) = -(1+\lambda_1) \quad (28)$$

Using $\tilde{u}_{B1} = \tilde{f}_1 e^{i\omega t}$, $\tilde{u}_{B2} = \tilde{f}_2 e^{i\omega t}$

The boundary conditions become

$$\tilde{f} = \tilde{f}_1 \quad \text{at} \quad y = \varepsilon \quad (29)$$

$$\frac{d\tilde{f}}{dy} = \alpha\sigma_1(\tilde{f}_1 - \frac{1+\lambda_1}{\sigma_1^2}) \quad \text{at} \quad y = \varepsilon \quad (30)$$

$$\tilde{f} = \tilde{f}_2 \quad \text{at} \quad y = 1 - \varepsilon \quad (31)$$

$$\frac{d\tilde{f}}{dy} = -\alpha\sigma_2(\tilde{f}_2 - \frac{1+\lambda_1}{\sigma_2^2}) \quad \text{at } y = 1 - \varepsilon \quad (32)$$

III. SOLUTION OF THE PROBLEM

Steady part:

Solving Eq.(21) subject to boundary conditions (22)-(25) we get the velocity field as

We get the velocity field as

$$\bar{u} = C_1 + C_2 e^{(1+\lambda_1)y} + y \quad (33)$$

The slip velocities are $\bar{u}_{B1}$ **and** $\bar{u}_{B2}$ **are given by**

$$\bar{u}_{B1} = \frac{(D_2E_2 - D_4E_1)}{(D_1D_4 - D_3D_2)}, \quad \bar{u}_{B2} = \frac{(D_3E_1 - D_1E_2)}{(D_1D_4 - D_3D_2)} \quad (34)$$

Unsteady part:

Solving equation (28) subject to boundary conditions (10) –(13), we get

$$\widetilde{f}(y) = Ae^{m_1 y} + Be^{m_2 y} - \frac{i}{\omega} \quad (35)$$

The unsteady part of the velocity is given by $\widetilde{u} = \widetilde{f}(y)e^{iwt}$

Separating real and imaginary parts, we get

$\widetilde{u} = (A_{12}Cos\,\omega t - B_{12}Sin\,\omega t) + i(B_{12}Cos\,\omega t - A_{12}Sin\,\omega t)$

The slip velocities are given by

$$\widetilde{u}_{B1} = \widetilde{f}_1 e^{i\omega t} = \left(\frac{Am_1 e^{m_1\varepsilon}}{\alpha\sigma_1} + \frac{Bm_2 e^{m_2\varepsilon}}{\alpha\sigma_2} + \frac{1+\lambda_1}{\sigma_1^{\,2}}\right)e^{i\omega t}$$

$$\widetilde{u}_{B2} = \widetilde{f}_2 e^{i\omega t} = \left(\frac{1+\lambda_1}{\sigma_2^{\,2}} - \frac{Am_1 e^{m_1(1-\varepsilon)}}{\alpha\sigma_2} - \frac{Bm_2 e^{m_2(1-\varepsilon)}}{\alpha\sigma_2}\right)e^{i\omega t}$$

5. Deductions

(i) Taking $k_1 = k_2 = k$ (i.e. $\sigma_1 = \sigma_2 = \sigma$) in equation (33) and (35) we obtain the velocity field for pulsatile flow of Jeffrey fluid between porous lined plates with equal permeability as follows:

Steady state velocity

$$\bar{u} = c_1 + c_2\, e^{(1+\lambda)Ry} + y \quad (36)$$

Unsteady part:

$$\tilde{u} = \left(A_1 e^{m_1 y} + A_2 e^{m_2 y} - \frac{i}{M^2}\right)e^{i\omega t} \quad (37)$$

(ii) When the permeabilities k_1 and k_2 tend to zero in equations (36) and (37) we obtain the velocity field for the pulsatile flow of the Jeffrey fluid in a channel bounded by rigid walls as follows

Steady part

$$\bar{u} = C_1 + C_2 e^{Ry} + y \quad (38)$$

The slip velocities $\tilde{u}_{B1}$ and $\tilde{u}_{B2}$ are zero.

Unsteady part

$$\tilde{u} = \left(A_1 e^{m_1 y} + A_2 e^{m_2 y} - \frac{i}{M^2}\right)e^{i\omega t} \quad (39)$$

The slip velocities $\tilde{u}_{B1}$ and $\tilde{u}_{B2}$ are zero.

Further with $\lambda \to 0$, the results (38) and (39) reduce the corresponding ones of Wang [9].

RESULTS AND DISCUSSION

From equation (33) we have calculated the steady part of the velocity as a function of y for different values of permeability parameter σ with fixed $\alpha = 0.5,\ R = 2,\ \lambda = 0.5,\ \varepsilon = 0.02$ and is shown in Fig.2. We observe that the velocity decreases with the increase in $\sigma = \frac{h}{\sqrt{k}}$. This may be due to the increase in the permeability 'k' of the upper and lower beds. Also, as the σ increases, the gap between the velocity profiles becomes smaller i.e. there is not much change in the velocity due to variation for large values of σ.

The variation of steady part of the velocity with y is calculated from equation (33) for different values of permeability parameter σ with fixed $\alpha = 0.1,\ R = 2,\ \lambda = 0.5,\ \varepsilon = 0.02$ and is shown in Fig.3. It is noticed that the velocity decreases with the increase in σ. From Fig.2 and Fig.3 it is noticed that the magnitude of the velocity increases with decrease in the slip parameter α. Also, as the σ increases, the gap between the velocity profiles becomes smaller.

The variation of $\bar{u}$ with y is calculated from equation (33) for different values of the Reynolds number R and for fixed $\alpha = 2,\ \sigma = 25,\ \lambda = 0.5,\ \varepsilon = 0.02$ and is shown in Fig.4. We observe that the velocity increases with the increase in R.

From equation (33) we have calculated the steady part of the velocity as a function of y for different values of Jeffrey parameter λ with fixed $\alpha = 2,\ \sigma = 25,\ R = 2,\ \varepsilon = 0.02$ and is shown in Fig.5. It is noticed that the velocity increases with the increase in the Jeffrey parameter λ. Similar behavior due to λ is noticed by Srinivas et al. (2008) and Hayat et al. (2008) for the peristaltic transport of viscous fluid in a flexible channel.

From equation (33) we have calculated the steady part of the velocity as a function of y for different values of the thickness of porous lined plate ε with fixed $\alpha = 2,\ \sigma = 25,\ R = 2,\ \lambda = 0.5$ and is shown in Fig.6. We observe that the velocity decreases with the increase in ε.

We have calculated the unsteady part of the velocity as the function of y from equation (35) for different values of wt with fixed $\alpha = 5,\ \sigma = 5,\ R = 0,\ \varepsilon = 0.02$, M=1, $\lambda = 1$ and is shown in Fig.7.We observe the velocity decreases with the increase in wt.

We have calculated the unsteady part of the velocity ($\tilde{u}$) as the function of y from equation (35) for different values of wt with fixed

$\alpha = 0.5,\ \sigma = 25,\ R = 10, \varepsilon = 0.02$, M = 1, $\lambda = 1$ and is shown in Fig.8.We observe the velocity decreases with the increase in wt. From Fig.7. and Fig.8. for fixed wt and α, as R increases the maximum velocity moves closure to the upper permeable bed.

We have calculated the unsteady part of the velocity ($\tilde{u}$) as the function of y from equation (35) and are shown in Figs.9 and 10. For fixed wt, the velocity decreases with the increment in σ. As R increases the maximum velocity moves closure to the upper permeable bed.

From equation (35) we have calculated the unsteady part of the velocity as a function of y for different values of Reynolds number R and Jeffrey parameter λ with fixed $\alpha = 5$, M=1, $\varepsilon = 0.02, wt = \frac{\pi}{4}$, $\sigma = 10$ and are shown in Fig.11. and Fig.12. It is noticed that the velocity increases with the increase in λ. As R increases the maximum velocity moves closure to the upper permeable bed.

The variation of $\tilde{u}$ with y is calculated from equation (35) for different values of the Reynolds number R and for fixed $\alpha = 2, \sigma = 25$, M=1, $\varepsilon = 0.02, wt = \frac{\pi}{4}$, $\lambda = 0.5$ and is shown in Fig.13. We observe that the velocity decreases with the increase in R.

From equation (35) we have calculated the unsteady part of the velocity as a function of y for different values of the thickness of porous lined plate ε with fixed $\alpha = 2, \sigma = 10$, M = 1, $R = 2, wt = \frac{\pi}{4}$, $\lambda = 0.5$ and is shown in Fig.14. We observe that the velocity decreases with the increase in ε.

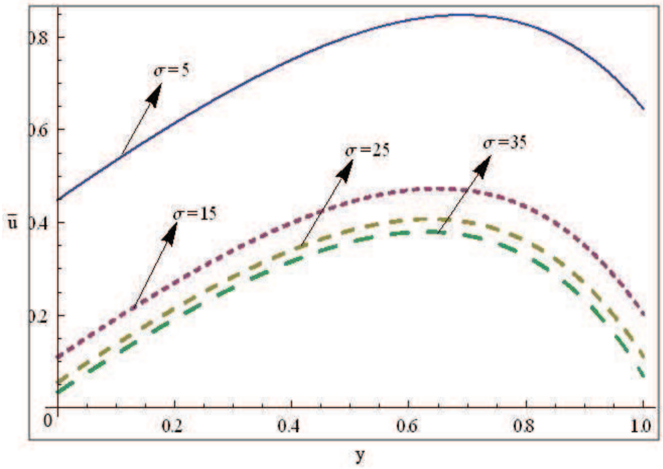

Fig. 2. Steady state velocity profiles for $\alpha = 0.5,\ R = 2,\ \lambda = 0.5$ and $\varepsilon = 0.02$

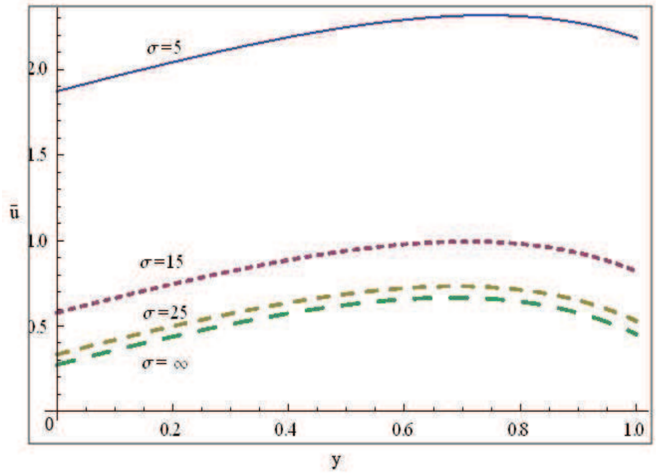

Fig. 3. Steady state velocity profiles for σ with fixed $\alpha = 0.1,\ R = 2,\ \lambda = 0.5$ and $\varepsilon = 0.02$

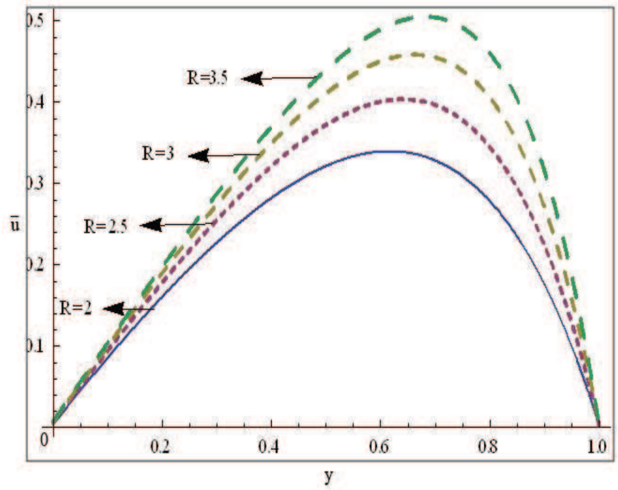

Fig..4. Steady state velocity profiles for R with fixed $\alpha = 2,\ \sigma = 25,\ \lambda = 0.5$ and $\varepsilon = 0.02$

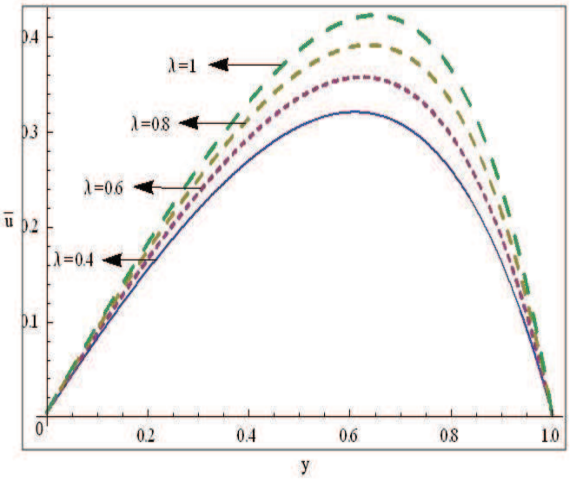

Fig..5. Steady state velocity profiles for λ with fixed $\alpha = 2, \sigma = 25, R = 2$ and $\varepsilon = 0.02$

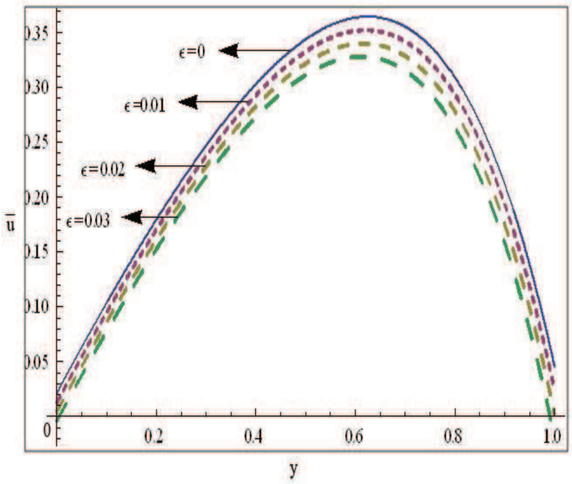

Fig. 6. Steady state velocity profiles for ε with fixed $\alpha = 2, \sigma = 25, R = 2$ and $\lambda = 0.5$

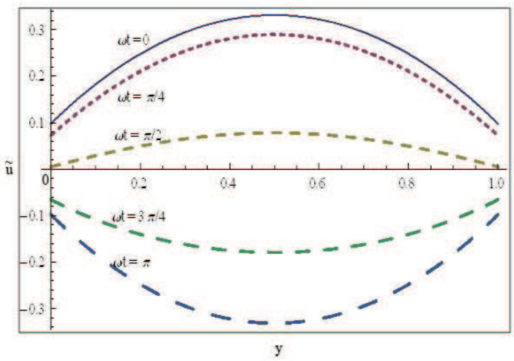

Fig.7. Unsteady state velocity profiles for ωt with fixed $\alpha = 5, \sigma = 5, R = 0$, $\varepsilon = 0.02$, M=1 and $\lambda = 1$

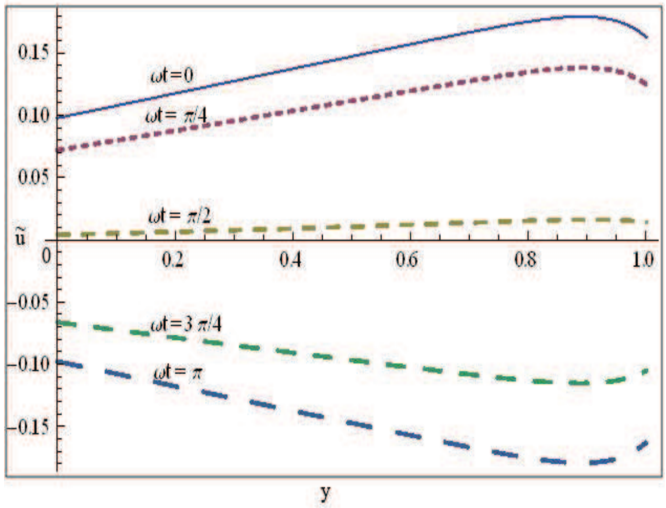

Fig.8. Unsteady state velocity profiles for ωt with fixed $\alpha = 0.5, \sigma = 25, R = 10$, $\varepsilon = 0.02$, M=1 and $\lambda = 0.5$

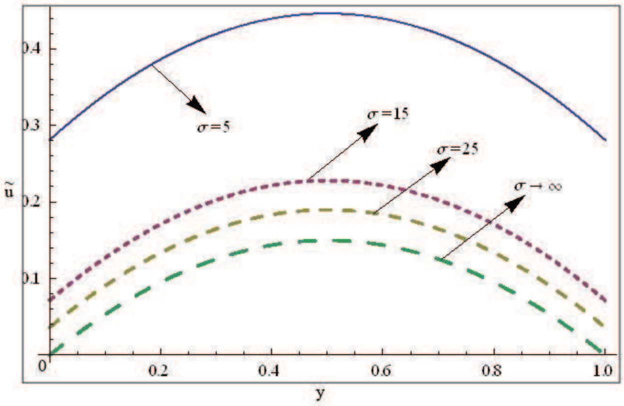

Fig.9. Unsteady state velocity profiles for σ with $\lambda = 0.5$

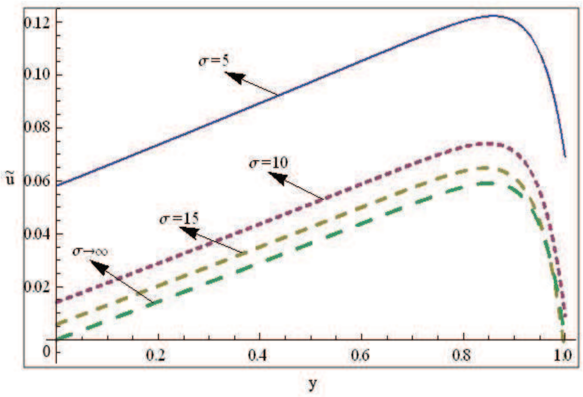

Fig. 10. Unsteady state velocity profiles for σ with $\lambda = 1$

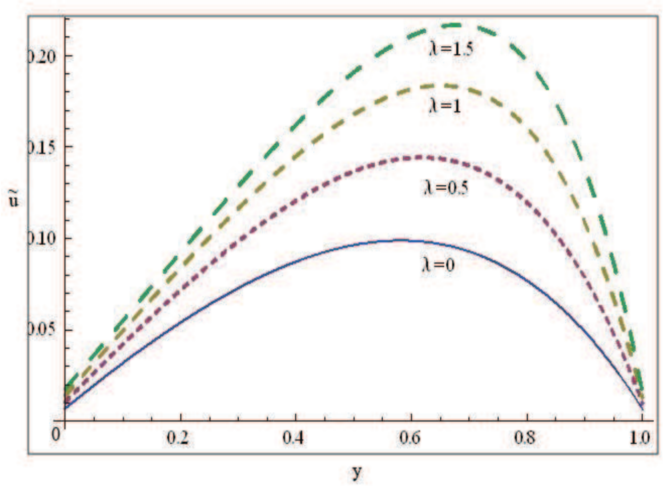

Fig. 11. Unsteady state velocity profiles for λ with fixed $\alpha = 5,\ R = 2$, M=1, $\varepsilon = 0.02, wt = \frac{\pi}{4}$ and $\sigma = 10$.

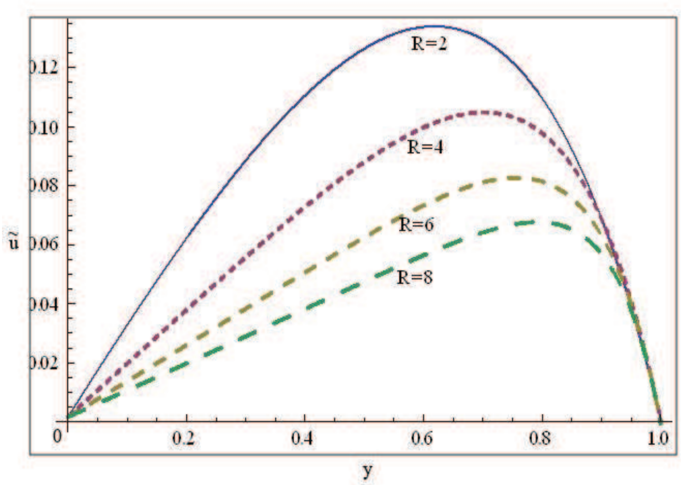

Fig. 13. Unsteady state velocity profiles for R with fixed $\alpha = 2,\ \sigma = 25$, M=1, $\varepsilon = 0.02, wt = \frac{\pi}{4}$ and $\lambda = 0.5$.

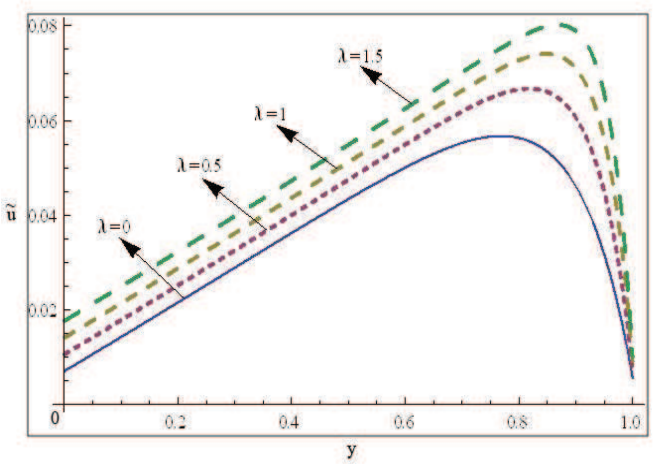

Fig. 12. Unsteady state velocity profiles for λ with fixed $\alpha = 5,\ R = 10$, M=1, $\varepsilon = 0.02, wt = \frac{\pi}{4}$ and $\sigma = 10$

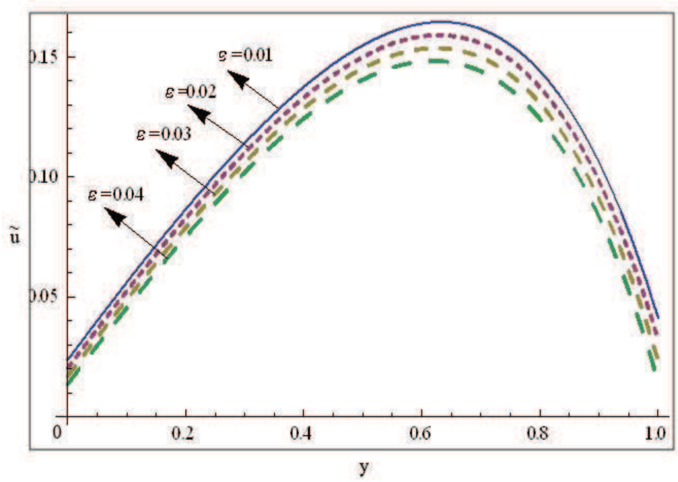

Fig. 14. Unsteady state velocity profiles for ε with fixed $\alpha = 2,\ \sigma = 10$, M=1, $R = 2, wt = \frac{\pi}{4}$ and $\lambda = 0.5$.

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