MHD effects on peristaltic flow of a Bingham fluid in a channel with permeable walls

V.Rama Chandraiah*, K.Chakradhar, R. Siva Prasad*****
* Department of Mathematics, Srinivasa Institute of Technology & Science, kadapa, A.P., INDIA
* *Department of Mathematics, Dr.Y.S.R. Engg. College of Technology, Kadapa, A.P., INDIA
*** Department of Mathematics, Sri Krishnadevaraya University, Anantapur, A.P., INDIA
E-Mail: ph.d9866368769@gmail.com

Abstract: MHD effects on peristaltic flow of a Bingham fluid in a channel with permeable walls is studied under long wavelength and low Reynolds number assumptions. This model can be applied to the blood flow in the sense that erythrocytes region and the plasma regions may be described as plug flow and non-plug flow regions. The effect of yields stress, Magnetic parameter, Darcy number and slip parameter on the pumping characteristics is discussed through graphs.

Keywords: Peristaltic transport, Magnetic parameter, Bingham fluid, permeable walls.

I. INTRODUCTION

Physiological fluids in animal and human bodies are in general, pumped by the continuous contraction and expansion of the ducts. These contractions and expansion are expected to be caused by peristaltic waves that propagate along the walls of ducts. In general during peristaltic action the fluid is pumped from lower pressure to higher pressure. Peristaltic transport occurs widely in the stomach, ureter, bile duct, small vessels etc. The principle of peristalsis is used by roller pumps for pumping fluids without being contaminated due to the contact with the pumping machinery. The initial work on peristalsis is done by using lab frame analysis. The important characteristics of peristaltic pumping namely trapping and reflux phenomenon are studied in detail by Shapiro et al [1] for the peristaltic flow of a viscous fluid through a tube and a channel.

In physiological peristalsis the pumping fluid cannot always be treated as a Newtonian fluid. Kapur [2] suggested several Non- Newtonian models for physiological flows. He made theoretical investigations regarding blood as a casson and Hershel- Bulkley fluids. Blair and Spanner [3] reported that blood obeys casson model for moderate shear rate flows. Lew et al [4] reported that chyme is a Non Newtonian material having plastic like properties. In view of this Sreenadh et al [5] studied the effect of yield stress on peristaltic pumping of Non- Newtonian fluids in a channel. The Non Newtinian fluids are Bingham and Herschel- Bulkley fluids. Vajravelu et al [6, 7] made a detailed study on the effect of yield stress on peristaltic pumping of Herschel-Bulkley fluid in an inclined tube and a channel. All these investigations are confined to hydromagnetic study of a physiological fluid obeying some yield stress model. It is reported that some physiological fluids like blood are conducting fluids. Motivated by this MHD effects on peristaltic flow of a Bingham fluid in a channel with permeable walls is investigated in this chapter under long wavelength and low Reynolds number assumptions. The expressions for the velocity field in the plug flow and non-plug flow regions, the pressure rise in the channel and the volume flow rate are obtained. The effects of the magnetic field, yield stress and amplitude ratio on the pumping characteristics are discussed.

II.MATHEMATICALFORMULATION

Consider the MHD effects on peristaltic pumping of a Bingham fluid in a channel with permeable walls, under long wave length and low Reynolds number assumptions (Fig 1).The flow between the permeable walls is governed by Navier-Stokes equations whereas the flow in the permeable beds is according to Darcy's law. The channel is of half-width a. The region between y=0 and y=y_0 is called plug flow region. In the plug flow region | $\tau_{yx}| \le \tau_0$ in the region between y=$y_0$and y=h, we have | $\tau_{yx}| > \tau_0$. The wall deformation is given by

$$H(X,t)=a+b\sin\frac{2\pi}{\lambda}(X-ct) \quad (1)$$

Where b is the amplitude, λ is the wave length and c is the wave speed.

Under the assumptions that channel length is an integral multiple of the wave length λ and the pressure difference across the ends of the tube is a constant, the flow becomes steady in the wave frame (x,y) moving with velocity c away from the fixed frame (X,Y). The transformation between these two frames is given by

X= X-ct; y= y; u(x,y)= U(X-ct,Y); v(x,y)=V(X-ct, Y);

Where (U, V) are velocity components in the laboratory frame and (u,v) are velocity components in the wave frame. In many physiological situations it is proved experimentally that the Reynolds number of the flow is very small. So, we assume that the wave length is infinite. So the flow is of Poiseuille type at each local cross-section.

We introduce the following non- dimensional quantities

$\bar{x}=\frac{x}{\lambda}$; $\bar{y}=\frac{y}{a}$; $\bar{z}=\frac{z}{a}$; $\bar{u}=\frac{u}{c}$; $\bar{w}=\frac{w}{c}$; $\delta=\frac{a}{\lambda}$; $\bar{t}=\frac{ct}{\lambda}$;
h = $\frac{H}{a}$; $\varphi=\frac{b}{a}$; $\tau_0=\frac{a\tau_c}{\mu c}$; $y_0=\frac{y_c}{a}$; $q=\frac{q}{ac}$;

In the equations governing the motion (dropping the bars) is

$$\frac{\partial}{\partial y}(\tau_0-\psi_{yy})-M^2(\psi_y+1)=-\frac{\partial p}{\partial x} \qquad (2)$$

$$0=\frac{\partial p}{\partial y} \qquad (3)$$

Where $M=B_0 a\sqrt{\frac{\sigma_e}{\mu}}$.

The non- dimensional boundary conditions are

$$\psi=0; \qquad \text{at } y=0 \qquad (4)$$

$$\psi_{yy}=\tau_0 \qquad \text{at } y=0 \qquad (5)$$

$$u=\psi_y=-1-\frac{\sqrt{Da}}{\alpha}\frac{\partial u}{\partial y} \qquad \text{at } y=h \qquad (6)$$

Where ψ is the stream function, Da $=\frac{k}{a^2}$, $\alpha=$ slip parameter and τ_0 is the yield stress.

SOLUTION OF THE PROBLEM

Solving equation (2) subject to the boundary conditions (4) to (6) we obtain the velocity as

$$u=\psi_y=-MC_1 Sin My+MC_2 Cos My-\frac{P}{M^2} \qquad (7)$$

Where, $C_1=(\frac{\tau_0-C}{M^2})$; $C_2=\frac{(-1+\frac{P}{M^2})+MC_1D_1}{MD_2}$; $P=(-\frac{\partial p}{\partial x}+M^2)$.

We find the upper limit of plug flow region using the boundary condition that

$\psi_{yy}=0$ at $y=y_0$ so we have $y_0=\frac{\tau_0}{P}$ (8)

Also by using the condition $\tau_{yx}=\tau_h$ at y=h

We obtain P= $\frac{\tau_h}{h}$

Hence $\frac{y_0}{h}=\frac{\tau_0}{\tau_h}=\tau$, $0<\tau<1$ (9)

Taking $y=y_0$ in equation (7) and using the relation (9) we get the velocity in plug flow region as

$$u_p=-MC_1 Sin My_0+MC_2 Cos My_0-\frac{P}{M^2} \qquad (10)$$

Integrating the equations (7) and (10) and using the conditions $\psi_p=0$ at y=0, $\psi=\psi_p$ at $y=y_0$.

we get stream function as

$$\psi=C_1 Cos My+C_2 Sin My+(\frac{-Py}{M^2}); \qquad (11)$$

The volume flux q through each cross- section on the wave frame is given by

$$q=\int_0^{y_0}u_p\,dy+\int_{y_0}^{h}u\,dy$$

$$=\frac{P}{M^2}(\frac{k_2-MhD_2}{MD_2})+s \qquad (12)$$

The instantaneous volume flow rate Q (x, t) in the laboratory frame between the centre line and the permeable wall is

$$Q(X,t)=\int_0^{H}U(X,Y,t)\,dY=\int_0^{h}(u+1)\,dy=q+h \qquad (13)$$

From equation (12), we have

$$\frac{dp}{dx}=\frac{M^3D_2(q-s)}{(k_2-MhD_2)}-M^2 \qquad (14)$$

The average volume flow rate $\overline{Q}$ over one wave period ($T=\frac{\lambda}{c}$) of the peristaltic wave as

$$\overline{Q}=\frac{1}{T}\int_0^T Q(X,t)dt=q+1 \qquad (15)$$

Integrating the equation (14) with respect to x over one wave length, we get the pressure rise over one cycle of the wave as

$$\Delta p=-\int_0^1\{\frac{M^3D_2(q-s)}{(k_2-MhD_2)}-M^2\}dx \qquad (16)$$

Where $h(x)=1+\phi\sin 2\pi x$

The time average flux at zero pressure rise (drop) is denoted by $\overline{Q_0}$ and the pressure rise required to produce zero average flow rate is denoted by Δp_0 so we have

$$\Delta p_0=-\int_0^1\{\frac{M^3D_2(-1-s)}{(k_2-MhD_2)}\}dx \qquad (17)$$

The dimensionless friction force F at the wall across one wave length is given by

$$F=\int_0^1 h(-\frac{dP}{dx})dx=-\int_0^1 h\{\frac{M^3D_2(q-s)}{(k_2-MhD_2)}-M^2\}dx \qquad (18)$$

III. DISCUSSION OF THE RESULT

In order to see the effect of various pertinent parameters such as the yield stress ($\bar{\tau}$), amplitude ratio ($\emptyset$), Darcy number (Da), Hartmann number (M) and slip parameter (α) on pumping characteristics we have plotted Figures 2 – 11.

The variation of pressure rise with time averaged flow rate is calculated from equation ((16)), for different amplitude ratios($\emptyset$) and is shown in figure (2) for fixed α = 2; Da=0.005; M=0.25; τ=0.6. We observe that the higher the amplitude ratio, the greater the pressure rise against which the pump works. For a given Δp , the flux $\bar{Q}$ for Bingham fluid depends on yield stress and it increases with increasing $\emptyset$.

From equation ((16)), we have calculated the pressure difference as a function of $\bar{Q}$ for different values of τ is shown in figure (3) for fixed values α = 2; Da=0.005; M=0.25; $\emptyset$=0.6. It is observed that for a Bingham fluid, the peristaltic wave passing over the channel wall pumps against more pressure rise (Δp) compared to Newtonian fluid. This type of behavior may be due to the presence of plug flow in Bingham fluid. Further, we observe that there is no difference in flux for Bingham fluid and Newtonian fluids for free pumping case ($\Delta p = 0$). For a given Δp the flux $\bar{Q}$ for a Bingham fluid depends on yield stress and it increases with increasing yield stress. From equation ((16)), we have calculated the pressure difference as a function of $\bar{Q}$ for different values of Darcy number Da, for fixed α = 2; M=0.25; τ=0.6; $\emptyset$=0.6 and is shown in figure (4), We observe that for a given Δp, the flux $\bar{Q}$ depends on Darcy number and it decreases with increasing Darcy number Da. For free pumping the flux $\bar{Q}$ is constant and it is independent of Da.

The variation of pressure rise with time averaged flow rate is calculated from ((16)), for different values of α and is shown in figure (5) for fixed Da=0.005; M=0.25; τ=0.6; $\emptyset$=0.6. We observe that the longer the slip parameter, the greater the pressure rise against which the pump works. For a givenΔp, the flux $\bar{Q}$ increases with increasingα. For a given flux$\bar{Q}$, the pressure difference ΔP increases with increasingα.

The variation of pressure rise with time averaged flow rate is calculated from ((16)), for different values of M and is shown in figure (6) for fixed α = 2; Da=0.005; τ=0.6; $\emptyset$=0.6. We observe that the larger the Hartmann parameter (M), the smaller the pressure rise against which the pump works. For a givenΔp, the flux $\bar{Q}$ decreases with increasing$M$. For a given flux$\bar{Q}$, the pressure difference Δp decreases with increasing$M$.

From equation (18), we have calculated the frictional force as a function of $\bar{Q}$ for different yield stress (τ), amplitude ratio ($\emptyset$), Darcy number (Da), Hartmann number (M) and slip parameter (α) and it is observed that the frictional force F has the opposite behavior compared to pressure rise (Δp) and is depicted in figures (7-11).

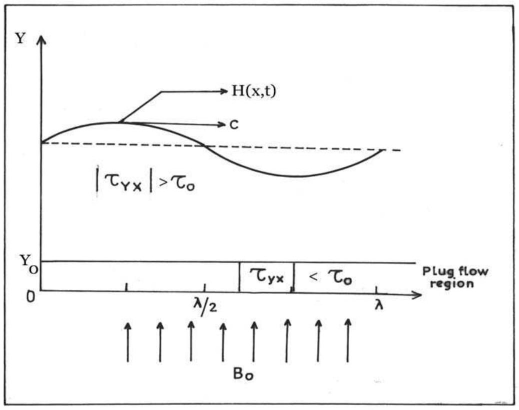

Fig. 1: Physical model for Bingham fluid in a channel with permeable walls.

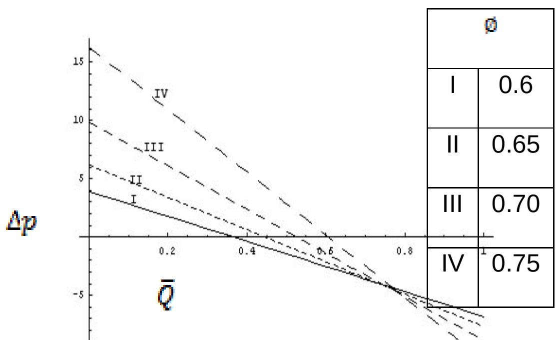

Fig: 2. The variation of Δp with $\bar{Q}$ for the different values of ø for fixed α = 2; Da=0.005; M=0.25; τ=0.6;

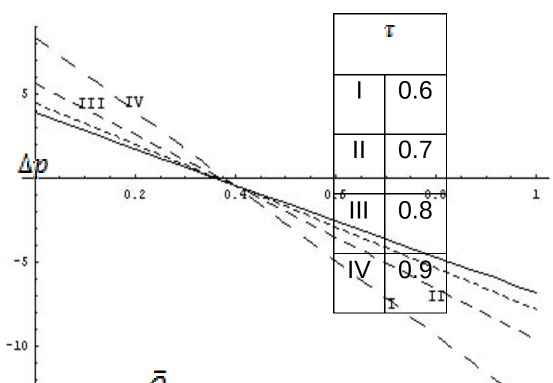

Fig: 3.The variation of Δp with $\bar{Q}$ for the different values of τ for fixed α = 2; Da=0.005; M=0.25; ø=0.6;

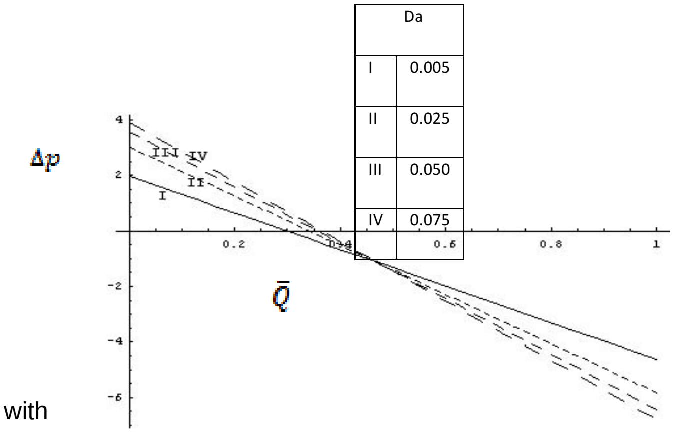

Fig: 4. The variation of Δp with $\bar{Q}$ for the different values of Da for fixed α = 2; M=0.25; τ=0.6; ø=0.6;

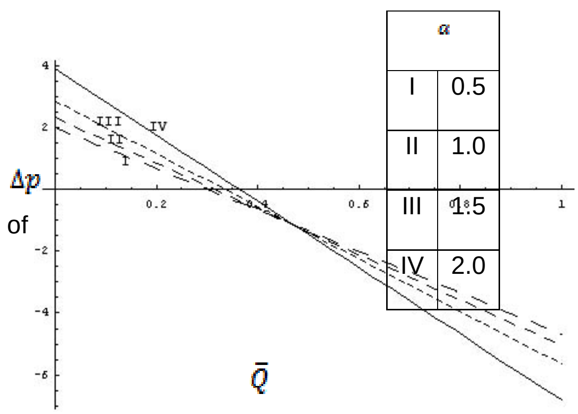

Fig: 5. The variation of Δp with $\bar{Q}$ for the different values of α for fixed Da=0.005; M=0.25; τ=0.6; ø=0.6;

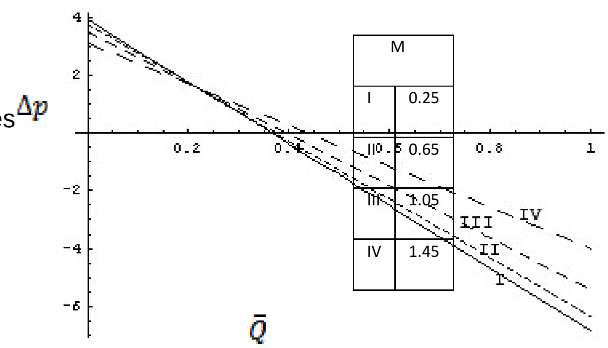

Fig: 6. The variation of Δp with $\bar{Q}$ for the different values of M for fixed α = 2; Da=0.005; τ=0.6; ø=0.6;

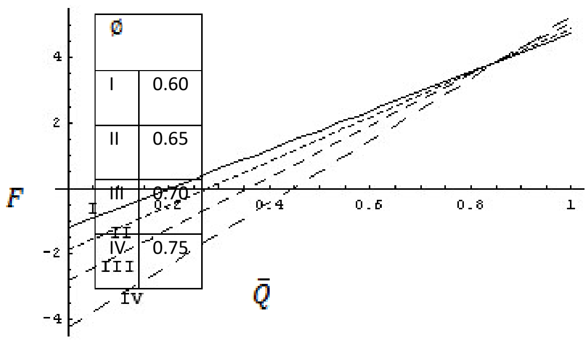

Fig: 7.The variation of F with $\bar{Q}$ for the different values of ϕ for fixed α = 2; Da=0.005; M = 0.25; τ=0.6;

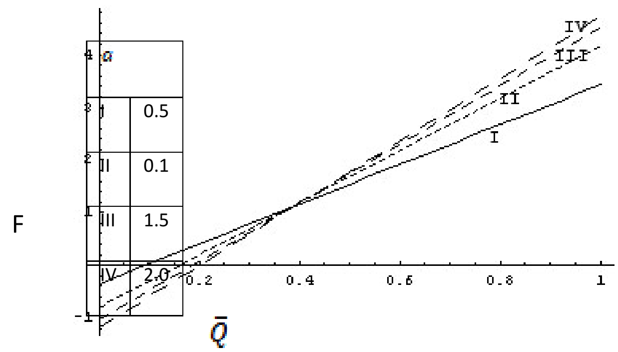

Fig: 10. The variation of F with $\bar{Q}$ for the different values of α for fixed Da=0.005; M=0.25; τ=0.6; ϕ=0.6;

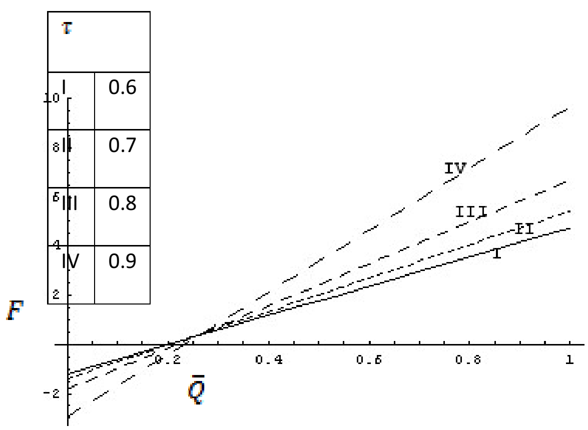

Fig: 8. The variation of F with $\bar{Q}$ for the different values of τ for fixed α = 2; Da=0.005; M=0.25; ϕ=0.6;

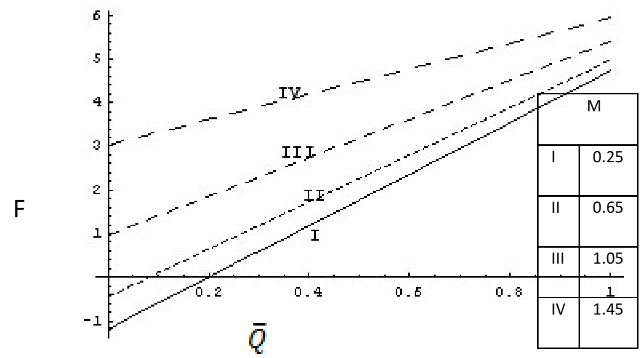

Fig: 11. The variation of F with $\bar{Q}$ for the different values of M for fixed α = 2; Da=0.005; τ=0.6; ϕ=0.6;

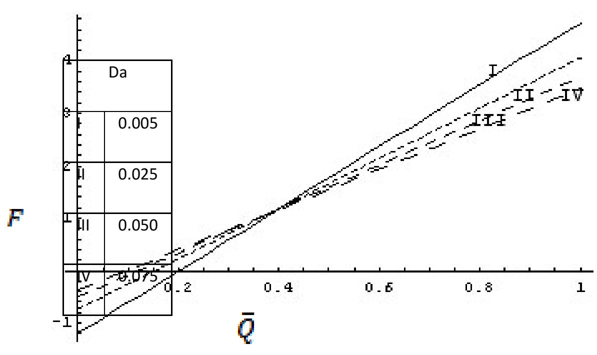

Fig:9. The variation of F with $\bar{Q}$ for the different values of Da for fixed α = 2; M=0.25; τ=0.6; ϕ=0.6;

REFERENCES

[1] Shapiro, A.H., Jaffrin, M.Y and Weinberg, S.L. Peristaltic pumping with long wavelengths at low Reynolds number, J. Fluid Mech. 37(1969), 799-825.

[2] Kapur, J.N. Mathematical Models in Biology and Medicine, Affiliated East-West Press Private Limited, New Delhi, 1985.

[3] G.W.S. Blair, D.C. Spanner, An Introduction to Bioreheology, Elsevier, Amsterdam, 1974.

[4] Lew, H.S., Fung, Y.C. and Lowenstein, C.B. Peristaltic carrying and mixing of Chyme in the small intestine (An analysis of a mathematical model of peristaltic of the small intestine), J. Biomech., 4 (1971), 297-315.

[5] Sreenadh, S., Narahari, M. and Ramesh Babu, V. Effect of yield stress on peristaltic pumping of Non-Newtonian fluids in a channel, International Symposium on advances in fluid Mechanics, UGC-Centre for Advanced Studies in Fluid mechanics, Bangalore University, June-2004, 234-247.

[6] Vajravelu, K., Sreenadh, S. and Ramesh Babu, V. Peristaltic transport of a Hershel-Bulkley fluid in an inclined tube, Int. J. Non linear Mech. 40(2005) 83-90.

[7] Vajravelu, K. Sreenadh, S. and Ramesh Babu, V. Peristaltic pumping of a Hershel-Bulkley fluid in a channel, Appl. Math. and Computation, 169(2005), 726-735.